The Amortization Method of Loan Payment

The amortization method is the most common method of loan repayment. The fundamental principle behind it is simple. When a payment is made, it must be first applied to pay interest due and then any remaining part of the payment is applied to pay principal.

Consider a loan for 30000$ with level payments to be made at the end of each year for 5 years at an annual rate of 8%.

Level loan payment can be calculated as:

$$30000 = Ra_{\overline{5}|}$$
$$R = 7513.69$$

Payment 1. Beginning Balance = 30,000
Interest due = $30,000(.08) = 2,400$
Payment made = 7513.69
Interest paid = 2400
Principal paid = $7,513.69 - 2,400.00 = 5,113.69$
Balance after payment = $30,000 - 5,113.69 = 24,886.31$

Payment 2. Beginning Balance = 24,886.31
Interest due = $24,886.31(.08) = 1,990.90$
Payment made = 7513.69
Interest paid = 1,990.90
Principal paid = $7,513.69 - 1,990.90 = 5,522.79$
Balance after payment = $24,886.31 - 5,522.79 = 19,363.52$

```
amortization_table<-function(Loan,n,m,i){ j=((1+(i/m))^m)-1 #converting of nominal interest rate to eff
term=n*m
pv_imm_ann = function(a, i, n) {
x = 0
r=1/(1+i)
for(i in 1:n) x = x + a * r^(i)
return(x)
}

P<-solve(pv_imm_ann(1,j,term),Loan)

interest = principal = balance =payment= vector("numeric", term)

 # calculate amortization schedule
outstanding_principal = Loan
for (i in 1:term) {

pymnt= P
intr = outstanding_principal * j
prnp = P - intr
outstanding_principal = outstanding_principal - prnp

payment[i]  =pymnt
interest[i]  = intr
```

```
principal[i] = prnp
balance[i] = outstanding_principal
}

schedule<-data.frame(month = 1:term,payment, interest, principal, balance)
schedule<-round(schedule,3)
return(format(schedule,scientific=F))

}

amortization_table(30000,5,1,0.08)
```

```
##   month  payment interest principal   balance
## 1     1 7513.694 2400.000  5113.694 24886.306
## 2     2 7513.694 1990.905  5522.789 19363.517
## 3     3 7513.694 1549.081  5964.612 13398.905
## 4     4 7513.694 1071.912  6441.781  6957.124
## 5     5 7513.694  556.570  6957.124     0.000
```

Now we can summarize the amortization using the following notation. For loan with periodic interest rate i,

Loan payment at time k:Pmt_k
Loan balance after Pmt_k is made:Bal_k
Principal paid in period k :$PRin_k$

Note that the loan amount is Bal_0. For $k \geq 1$ the amortization method is described by the recursive relations:

Interest paid in Pmt_{k+1} : $Int_{k+1} = i(Bal_k)$
Principal paid in Pmt_{k+1} : $PRin_{k+1} = Pmt_{k+1} - i(Bal_k)$

Formulas for Level Payment Loan Amortization

It can be shown that for a level payment loan with payment P,
Interest paid in Pmt_t : $Int_t = P(1 - v^{n-t+1})$
Principal paid in Pmt_t : $PRin_t = Pv^{n-t+1}$